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THE STRUCTURE OF THE DOPPLER-DIFFERENCE SIGNAL AND THE ANALYSIS--ETC(U)

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SUMMARY

The Doppler-difference optical arrangement is nowadays the one most commonly used for fluid flow investigations with a laser anemometer. In this review, the essential characteristics of the Doppler-difference signal and of the associated autocorrelation function are considered for both laminar and turbulent flows. Experimental conditions under which it is known that the autocorrelation function can be analysed to yield reliable estimates of mean velocity and turbulence intensity are specified.

The various data-reduction methods which have been proposed are classified and briefly reviewed. It is shown that unknown flows having relatively high levels of turbulence can only be treated successfully if there are very many fringes within a beam diameter, or if frequency-shifting techniques are used to achieve an equivalent effect. The estimation of turbulence intensity in low-turbulence flows also presents special difficulties and a procedure which uses information already available in the transform plane is described.

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1 INTRODUCTION

We consider in this review the problem of extracting information concerning the mean value and the fluctuations of some specified component of velocity of a fluid from the autocorrelation function of the intensity of the radiation scattered from a localised region, the measuring volume, by small particles embedded in the fluid. It is assumed that the scatterer follows the bulk motion of the fluid with perfect fidelity. We shall be concerned with the output of an ideal autocorrelator operating on the signals obtained with a Doppler-difference optical system which, because of its ease of alignment, inherent stability and the efficiency of incoherent light-collection, has become the most widely-used laser velocimetry arrangement in fluid-mechanical applications. Detailed discussions of the autocorrelation functions encountered in other forms of light-beating experiments, such as those using homodyne or heterodyne techniques, can be found in Ref 1. We shall not consider the form of the output obtained from Doppler-difference signals with a clipping correlator. (The formulae given here should remain valid, however, if the clipping level is set equal to zero and the number of counts per sample time is very much less than one.)

2 THE SCATTERED INTENSITY AND ITS AUTOCORRELATION FUNCTION IN LAMINAR AND TURBULENT FLOWS

We assume that the light source is a linearly polarised laser operating in the fundamental (Gaussian) TEM_{00} mode. Coherence requirements are very modest and restriction to a single longitudinal mode of operation is usually unnecessary. We assume further that the motion of the scattering particle follows that of the fluid with perfect fidelity; in aerodynamic applications sub-micron particles are usually necessary, and are always advisable for experiments on gaseous flows. It will be shown later that a small angle of intersection between the incident beams is an important feature of a system designed for use on a completely general turbulent flow. Under these conditions it is a reasonable approximation to assume that for equal incident beam-powers, the scattered light intensities in any given direction are equal. For larger angles of intersection the depth of modulation of the Doppler-difference signal may depend significantly on the optical geometry, and account would have to be taken of the effect in any more general theory.

We suppose then that the scatterer radiates with equal efficiency from the two incident beams. We allow for the fact that these may not be perfectly balanced by introducing the quantity ρ , the ratio of their amplitudes. We shall assume, however, that the beams have the same characteristic radius r_0 ;

this is the radial distance at which the amplitude of the electric field has fallen to e^{-1} of its axial value. If θ is the angle of intersection of the beams, the output of a square-law detector (such as a photomultiplier tube) on which the scattered radiation falls will take the form[2]

$$I(x,y,z) = I_0 \exp\left[\left\{-\frac{2}{r_0^2}\left(x^2 \sin^2\frac{\theta}{2} + y^2\cos^2\frac{\theta}{2} + z^2\right)\right\}\right]$$
$$\times \left[\exp\left(\frac{4}{r_0^2}xy \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) + \rho^2 \exp\left(-\frac{4}{r_0^2}xy\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\right.$$
$$\left. + 2\rho\cos\left(\frac{4\pi}{\lambda} y \sin\frac{\theta}{2}\right)\right] \qquad (1)$$

where (x,y,z) are the coordinates of the scatterer's position. I_0 is a constant of proportionality which includes other constants such as the collecting efficiency. If the particle is moving with velocity (v_x,v_y,v_z) and at time $t = 0$ is at some general point (ξ_0,η_0,ζ_0) on its trajectory (see Fig 1), then

$$x = \xi_0 + v_x t$$
$$y = \eta_0 + v_y t$$
$$z = \zeta_0 + v_z t \quad .$$

$I(x,y,z)$ is now a function of time, $I(t)$. The frequency of the cosine term in equation (1) is seen to be v_y/s, where s is the Doppler-difference 'fringe' spacing, defined in the usual way:

$$s = \lambda\Big/\left(2\sin\frac{\theta}{2}\right) \quad .$$

$I(t)$ represents the analogue signal which would be obtained if the scattered radiation were sufficiently intense. At very low signal-levels it represents instead the probability of photon detection by the photocathode.

The output of an autocorrelator operating on such a signal will be proportional to the function

$$G(\tau) = \int_{-\infty}^{\infty} I(t)I(t+\tau)dt \quad .$$

This expression, relating to a laminar flow, has been evaluated for a general optical geometry in Ref 2. The result is complicated, but simplified forms yielding more insight into its physical significance have been obtained for particular cases and are presented elsewhere in these proceedings[3].

A formula applicable to three-dimensional turbulence was also derived in Ref 2 by making the simplifying assumption that the beam intersection angle is very small, so that the measuring volume becomes approximately cylindrical. Except in the case of fully forward or fully backward scattering, the detector can usually be arranged to provide a well-defined cut-off in the direction of the beams. We summarise here the main feature of the subsequent argument.

Equation (1) can be rewritten as

$$\begin{aligned} f(t) \equiv \frac{I(t)}{I_0} = \left[\exp\left(-\frac{2z^2}{r_0^2}\right)\right]\Bigg[&\exp\left\{-\frac{2}{r_0^2}\left(x \sin\frac{\theta}{2} - y \cos\frac{\theta}{2}\right)^2\right\} \\ &+ \rho^2 \exp\left\{-\frac{2}{r_0^2}\left(x \sin\frac{\theta}{2} + y \cos\frac{\theta}{2}\right)^2\right\} \\ &+ 2\rho \exp\left\{-\frac{2}{r_0^2}\left(x^2 \sin^2\frac{\theta}{2} + y^2 \cos^2\frac{\theta}{2}\right)\right\} \cos\frac{2\pi}{s}y\Bigg] \quad . \quad (2) \end{aligned}$$

One usually possesses some *a priori* knowledge of the mean flow direction and we will assume that the beams are oriented in such a direction that transits for which v_x is large are relatively uncommon. The angle of intersection is now supposed so small that for the great majority of transits the quantity $x \sin \theta/2$ is negligible compared with $y \cos \theta/2$. (As an example, suppose the fringe size is $10\ \mu m$ and $\lambda = 0.5\ \mu m$. Then $\theta \simeq \lambda/s \simeq 0.05 \simeq 3^o$, and $\sin \theta/2 = 0.025$. If the detector aperture is adjusted to make $x_{max} = 4r_0$, say, we have

$$\left|\left(x \sin\frac{\theta}{2}\right)\right|_{max} = 0.1 r_0 \Big) .$$

Then a good approximation to equation (2) is

$$f(t) = \left[\exp\left\{-\frac{2}{r_0^2}(y^2 + z^2)\right\}\right]\left[1 + \rho^2 + 2\rho \cos \frac{2\pi}{s} y\right]$$

$$= (1 + \rho^2)\left[\exp\left\{-\frac{2}{r_0^2}(y^2 + z^2)\right\}\right]\left(1 + m \cos \frac{2\pi}{s} y\right) , \tag{3}$$

where m is the Michelson fringe visibility:

$$m = \frac{2\rho}{1 + p^2} .$$

Further simplification is possible. Suppose the point of closest approach to the x-axis has coordinates (x_0, y_0, z_0) . The vector joining this point to the x-axis is normal to the particle trajectory and lies in the direction

$$y_0 \underset{\sim}{j} + z_0 \underset{\sim}{k}$$

Hence

$$y_0 v_y + z_0 v_z = 0$$

and we find from equation (3)

$$f(t) = (1 + \rho^2)\left[\exp\left\{-\frac{2}{r_0^2}(y_0^2 + z_0^2)\right\}\right]$$

$$\times \left[\exp\left\{-2\frac{(v_y^2 + v_z^2)}{r_0^2} t^2\right\}\right]\left[1 + m \cos \frac{2\pi}{s}(y_0 + v_y t)\right] . \tag{4}$$

The autocorrelation function of this expression has been shown in Ref 2 to be, to a very close approximation ,

$$G(\tau) = \frac{\sqrt{\pi}}{2} r_0 (1 + \rho^2)^2 \left[\exp\left\{-\frac{4}{r_0^2}(y_0^2 + z_0^2)\right\}\right](v_y^2 + v_z^2)^{-\frac{1}{2}}$$

$$\times \exp\left\{-\frac{(v_y^2 + v_z^2)}{r_0^2}\tau^2\right\}\left[1 + \tfrac{1}{2} m^2 \cos\left(\frac{2\pi}{s} v_y \tau\right)\right] . \tag{5}$$

$f(t)$ and $G(\tau)$ represent the function of time and the autocorrelation function which would be obtained under conditions of laminar flow, or from a single strong scatterer. We remark here that they contain similar exponential multiplying factors, functions of t and τ respectively, so that high-pass filtering of either $f(t)$ or $G(\tau)$ in order to obtain an estimate of v_y from the cosine term would introduce similar errors.

The notation will be changed at this point to conform with a common aerodynamic convention. The velocity component normal to the fringe planes, v_y, will be denoted by u and the component perpendicular to the plane of the beams, v_z, by v. The third orthogonal component will be denoted by w. Then equation (5) becomes

$$G(\tau) = \frac{a_0}{\sqrt{u^2 + v^2}} \left[\exp\left\{ -\frac{(u^2 + v^2)\tau^2}{r_0^2} \right\} \right] \left(1 + \tfrac{1}{2}m^2 \cos \frac{2\pi u\tau}{s} \right) , \tag{6}$$

where the various constants have been included under the symbol a_0. If $G(\tau)$ represents a summation over a number of transits, a_0 will be understood to involve an average value of the quantity $(y_0^2 + z_0^2)$, in an obvious sense.

In turbulent flow, successive particles cross the measuring volume generally with different velocities, and the resulting autocorrelation function will be an integral of the laminar form for $G(\tau)$ of (6) over the probability density function $p(u,v,w)$ which characterises the turbulence. The fact that the scattering volume is approximately cylindrical can be exploited[2] to show that the composite autocorrelation function - the output of the correlator in fully turbulent conditions - becomes

$$H(\tau) = a_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(u,v,w) \times \left[\exp\left\{ -\frac{(u^2 + v^2)\tau^2}{r_0^2} \right\} \right] \left(1 + \tfrac{1}{2}m^2 \cos \frac{2\pi u\tau}{s} \right) du\,dv\,dw . \tag{7}$$

where a_1 now includes constants depending on the numbers of particles, the optical geometry and the duration of the experiment. In deriving (7) it was assumed that the particles are completely randomly distributed in an incompressible fluid, and that the dimensions of the measuring volume are small compared with the length scale of the turbulence. The dependence of the instantaneous rate at which transits occur on the fluid velocity leads to the disappearance in (7) of the factor $\sqrt{(u^2 + v^2)}$ appearing in (6).

3 ANALYSIS OF THE AUTOCORRELATION FUNCTION

3.1 Laminar flows

Here equation (6) applies and data-reduction can proceed by straightforward curve-fitting. Alternatively, the problem can be regarded as a special case of the more general one of estimating mean velocity in turbulent flow and a single computer programme can be designed to cover both conditions. Purely laminar flows are in any case rarely, if ever, encountered in practice and an estimate of the turbulence level present is usually wanted.

3.2 Turbulent flows

We assume that fluctuations may be occurring in all three components and note that integration over the w-component in (7) can be carried out immediately, since the kernel of the integral is independent of w .

By definition, the joint probability for the u- and v-components is

$$p_{uv}(u,v) = \int_{-\infty}^{\infty} p(u,v,w)\,dw$$

and hence

$$H(\tau) = a_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{uv}(u,v)\left[\exp\left\{-\frac{(u^2+v^2)\tau^2}{r_0^2}\right\}\right]\left(1 + \tfrac{1}{2}m^2 \cos\frac{2\pi u\tau}{s}\right) du\,dv \quad . \qquad (8)$$

In practice there will always be a nearly-constant (dc) level added to the signal, which arises mainly from background radiation present during the experiment. This is the value to which $G(\tau)$ will tend as τ tends to infinity and is normally included in the data-reduction procedures as another unknown variable whose magnitude has to be determined.

Equation (8) presents us with an impasse, for we cannot recover the two-dimensional distribution p_{uv} from an experimental function of a single variable. The problem centres essentially on the exponential term, since this factor couples both u- and v-components into the measured correlation function, and it is obvious that some extra information is required if progress is to be made at all. If a complete mathematical model for p_{uv} is available, data-reduction can be carried out by curve-fitting to $H(\tau)$. This will not be the case, however, in most problems of any real intrinsic interest (*ie* for the majority of experiments), and we now consider special cases, involving weaker assumptions, for which useful information about the velocity field can be recovered from the experimental function $H(\tau)$.

3.2.1 p_{uv} separable

We write

$$p_{uv}(u,v) \quad = \quad p_u(u)p_v(v) \quad .$$

Note that this implies that the stress term $\overline{u'v'}$ must be zero; primes denote the zero-mean fluctuating parts of the velocity components and capital letters their mean values:

$$u \;=\; U + u' \qquad\qquad v \;=\; V + v'$$

For, by definition

$$\begin{aligned}
\overline{uv} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} uvp_{uv}(u,v)\,du\,dv \\
&= \int_{-\infty}^{\infty} up_u(u)\,du \int_{-\infty}^{\infty} up_v(v)\,dv \\
&= UV \quad .
\end{aligned}$$

But

$$\begin{aligned}
\overline{uv} &= \overline{(U + u')(V + v')} \\
&= UV + \overline{u'v'} \quad .
\end{aligned}$$

Hence

$$\overline{u'v'} \;=\; 0$$

Equation (8) can be written

$$H(\tau) \;=\; a_1 \int_{-\infty}^{\infty} p_v(v)\, \exp\left(-\frac{v^2\tau^2}{r_0^2}\right) dv \int_{-\infty}^{\infty} p_u(u)\left[\exp\left(-\frac{u^2\tau^2}{r_0^2}\right)\right]\left(1 + \tfrac{1}{2}m^2 \cos\frac{2\pi u\tau}{s}\right)du \,. \qquad \ldots\ldots (9)$$

Both integrals are unknown functions of τ , so that progress cannot be made in determining the characteristics of either p_u or p_v without further assumptions.

(a) u- and v-components both normally distributed

We put

$$p_u(u) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left\{-\frac{(u-U)^2}{2\sigma_u^2}\right\}$$

where σ_u is the standard deviation of the fluctuations in u, with a similar expression for $p_v(v)$.

In this case the integrals can be evaluated explicitly, and we find, to a very good approximation,

$$H(\tau) = a_1\left[\exp\left\{-\frac{(U^2+V^2)\tau^2}{r_0^2}\right\}\right]\left[1 + \tfrac{1}{2}m^2\exp\left(-\frac{2\pi^2\sigma_u^2\tau^2}{s^2}\right)\cos\frac{2\pi u\tau}{s}\right] \tag{10}$$

for values of τ such that

$$\tau^2 \ll \frac{1}{2}\frac{r_0^2}{\sigma_u^2}, \quad \frac{1}{2}\frac{r_0^2}{\sigma_v^2} . \tag{11}$$

(With these restrictions, the result can also be shown to hold if p_{uv} is a bivariate normal distribution, so that a shear stress term can be accommodated[4]. In this case, p_{uv} is not separable.)

It is often more convenient to work in terms of the turbulence intensity η, the ratio of the standard deviation to the mean, rather than in terms of the standard deviation itself. For example, suppose we are using a 48-channel correlator operating at 50 ns per channel in an experiment on a supersonic flow where the mean velocity is 400 m s^{-1}. Then τ_{max} is 2.4 μs and for a beam diameter of 1 mm, the inequalities would require that both η_u and η_v are less than about 0.15 (15%) or so.

(b) Hermite polynomial expansion for $p_u(u)$: v-component normally distributed

This approximation[5] is intended for applications where the distribution of one velocity component only may be highly non-Gaussian and rests on the assumption that $p_v(v)$ is a Gaussian and that $p_u(u)$ can be written in the form of a Gram-Charlier expansion:

$$p_{uv}(u,v) = \frac{1}{2\pi\sigma_u\sigma_v} \exp\left\{-\frac{(u-U)^2}{2\sigma_u^2} - \frac{(v-V)^2}{2\sigma_v^2}\right\} \sum_{k=0}^{2L-1} A_k He_k\left(\frac{u-U}{\sigma_u}\right)$$

where $$A_k = \frac{1}{k!}\left\langle He_k\left(\frac{u-U}{\sigma_u}\right)\right\rangle$$

and He_k is the Hermite polynomial defined by the relation

$$He_k(z) = (-)^k \exp(\tfrac{1}{2}z^2) \frac{\partial^k}{\partial z^k}\left[\exp(-\tfrac{1}{2}z^2)\right] .$$

The choice of L in the upper limit $(2L-1)$ of the sum will depend on the amount of information required from the calculation. The first few polynomials have the following forms:

$$He_0(z) = 1 , \quad He_1(z) = 2z , \quad He_2(z) = 4z^2 - 2 ,$$

$$He_3(z) = 8z^3 - 12z , \quad He_4(z) = 16z^4 - 48z^2 + 12 .$$

The double integral can again be evaluated explicitly and, using again the approximations of (11), takes the form

$$H(\tau) = a_1\left[\exp\left\{-\frac{(U^2+V^2)\tau^2}{r_0^2}\right\}\right]\left[1 + \tfrac{1}{2}m^2 \exp\left(-\frac{2\pi^2\sigma_u^2\tau^2}{s^2}\right)\right]\sum_{k=0}^{L}(-)^k\left(\frac{2\pi\sigma_u\tau}{s}\right)^{2k}$$
$$\times\left[A_{2k}\cos\left(\frac{2\pi U\tau}{s}\right) + A_{2k+1}\left(\frac{2\pi\sigma_u\tau}{s}\right)\sin\left(\frac{2\pi U\tau}{s}\right)\right] . \quad (12)$$

The A_k are now determined by curve-fitting. Fig 2 of Ref 5 illustrates the way in which a much better approximation can be obtained to a bimodal form of $p_u(u)$ using the Gram-Charlier method than with the purely Gaussian model of the last section.

(c) Eigenfunction expansion for $p_u(u)$: v-component assumed negligibly small

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Here the correlation function reduces approximately to the form

$$H(\tau) = a_1 \int_{-\infty}^{\infty} p_u(u)\left[\exp\left(-\frac{u^2\tau^2}{r_0^2}\right)\right]\left(1 + \tfrac{1}{2}m^2 \cos\frac{2\pi u\tau}{s}\right)du \quad .$$

It is convenient at this point to introduce an important practical consideration. Because the kernel of the integral is an even function of u, any negative values of u which may occur will contribute to $H(\tau)$ in exactly the same way as equal positive ones. Hence if an undistorted form of $p_u(u)$ is to be recovered, the u-component must always be of the same sign; *ie* the flow must be non-reversing. (In addition, the analysis summarised in this section rests on Hilbert-Schmidt theory, which requires that the kernel be symmetric in u and τ.) Let us suppose then that u is always positive, so that the lower limit of the integral can be set equal to zero:

$$H(\tau) = a_1 \int_{0}^{\infty} p_u(u)\left[\exp\left(-\frac{u^2\tau^2}{r_0^2}\right)\right]\left(1 + \tfrac{1}{2}m^2 \cos\frac{2\pi u\tau}{s}\right)du \quad . \tag{13}$$

The procedure now is to try to find a set of functions ϕ_i and factors λ_i, eigenfunctions and eigenvalues, which satisfy the associated Fredholm equation

$$\int_0^{\infty} \phi_i(u)K(u\tau)\,du = \lambda_i\phi_i(\tau)$$

where $$K(u\tau) = \left[\exp\left(-\frac{u^2\tau^2}{r_0^2}\right)\right]\left(1 + \tfrac{1}{2}m^2 \cos\frac{2\pi u\tau}{s}\right) \quad .$$

Suppose also that the ϕ_i form a complete orthogonal set. Then any arbitrary function, such as $p_u(u)$, can be expanded in a series of the form

$$p_u(u) = \sum_{j=0}^{\infty} c_j\phi_j(u)$$

where the c_j are unknown coefficients which have to be determined. Substituting this expression into the equation for $H(\tau)$ and proceeding formally, we have 680

$$H(\tau) = a_1 \int_0^\infty \left[\sum_{j=0}^\infty c_j \phi_j(u) \right] K(u\tau)\,du$$

$$= a_1 \sum_{j=0}^\infty c_j \lambda_j \phi_j(\tau) \quad .$$

Using the orthogonality of the ϕ_i , we have

$$\int_0^\infty H(\tau)\phi_k(\tau)\,d\tau = a_1 \int_0^\infty \left[\sum_{j=0}^\infty c_j \lambda_j \phi_j(\tau) \right] \phi_k(\tau)\,d\tau$$

$$= a_1 c_k \lambda_k$$

so that

$$c_k = \frac{1}{a_1 \lambda_k} \int_0^\infty H(\tau)\phi_k(\tau)\,d\tau \quad ,$$

and finally

$$p_u(u) = \frac{1}{a_1} \sum_{j=0}^\infty \frac{1}{\lambda_j} \phi_j(u) \left[\int_0^\infty H(\tau)\phi_j(\tau)\,d\tau \right] \quad . \tag{14}$$

A complete set of eigenfunctions and their associated eigenvalues for the Doppler-difference kernel has been found. They are discussed in detail and their application to experimental data described in another contribution to these proceedings[6].

3.2.2 p_{uv} not separable

Consider again equation (8):

$$H(\tau) = a_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{uv}(u,v) \left[\exp\left\{ - \frac{(u^2 + v^2)\tau^2}{r_0^2} \right\} \right] \left(1 + \tfrac{1}{2} m^2 \cos \frac{2\pi u\tau}{s} \right) du\,dv \quad .$$

We discuss two important cases, distinguished by the behaviour of the exponential term.

(a) Exponential term approximately unity

If there are sufficient fringes across the beam ($r_o \gg s$) the exponential term will be almost unity over the observed length of the autocorrelation function and we obtain immediately the approximation

$$H(\tau) \doteq a_2 \int_{-\infty}^{\infty} p_u(u) \cos \frac{2\pi u\tau}{s} \, du + a_1$$

since

$$p_u(u) \triangleq \int_{-\infty}^{\infty} p_{uv}(u,v) dv \quad .$$

Note that it is unnecessary to make assumptions now about the shape of the function p_{uv} and that high turbulence levels in the v-component will not affect the result. Again we remark that the correct form for the distribution $p_u(u)$ can only be obtained if the u-component is of constant sign. Here we assume that, if necessary, a constant known bias u_m is imposed in order to make $u + u_m$ positive for each particle transit. This can be achieved by introducing a frequency-shifting device into one or both beams; see for example Ref 7 for a review of available methods. We can then without loss of generality again replace the lower limit of the integral by zero:

$$H(\tau) = a_2 \int_{0}^{\infty} p_u(u) \cos \frac{2\pi u\tau}{s} \, du + a_1 \quad . \tag{15}$$

Hence $p_u(u)$ can be recovered by performing a simple Fourier cosine transform on the data, after removal of the additive background constant a_1 . The latter can easily be carried out automatically by the data-reduction programme - high accuracy is not necessary.

The requirement that $r_0 \gg s$ implies a lower limit to the spatial resolution, characterised by the beam diameter $2r_0$, which can be achieved in high-speed flows. With a minimum sample time of 50 ns and beams of not more than 1 mm diameter, the condition cannot be satisfied, for all practical purposes, in most supersonic applications. (In nearly laminar conditions frequency-shifting could conceivably be used to bring the effective velocity down.) However, for low-speed flows it is very often possible to arrange the experiment appropriately. Note also that if the speed is sufficiently low, frequency-shifting methods can now be used to accommodate very high levels of turbulence[8], for which flow reversals will occur from time to time, or to achieve fine spatial resolution.

(b) <u>Exponential term varying significantly: narrow distributions of the velocity components</u>

These conditions are met with, for example, in supersonic boundary layers at low Mach numbers[2]. In this case, since u and v are assumed to lie within a narrow range, the exponential term in (8) varies much more slowly within the integral, for a given value of τ, than the cosine term and can be replaced to a good approximation by the value it takes when u and v are given their mean values, U and V. It can then be taken outside the integral and the integration over the v-component again carried out, yielding the approximate expression

$$H(\tau) = a_1 \left[\exp\left\{ - \frac{(U^2 + V^2)\tau^2}{r_0^2} \right\} \right] \int_{-\infty}^{\infty} p_u(u) \left(1 + \tfrac{1}{2} m^2 \cos \frac{2\pi u \tau}{s} \right) du \quad .$$

Since the turbulence level is low the lower limit can be replaced by zero, and finally we have

$$H(\tau) = a_1 \left[\exp\left\{ - \frac{(U^2 + V^2)\tau^2}{r_0^2} \right\} \right] \int_{0}^{\infty} p_u(u) \left(1 + \tfrac{1}{2} m^2 \cos \frac{2\pi u \tau}{s} \right) du \quad . \tag{16}$$

If the mean flow direction is not known, two fringe systems can be used to obtain estimates of U and V. For narrow nearly symmetric velocity distributions, the positions of the peaks in the transform plane are barely affected by the exponential term and a linear approximation to it will provide good initial values for U and V. The computer programme can be designed to determine r_0 from these estimates, together with any residual background contribution to the correlation function. The magnitude of the exponential term can now be calculated for each value of τ and a modified form of $H(\tau)$ derived from which the damping effect of the exponential has been eliminated. A second Fourier transform will now give an improved estimate for $p_u(u)$ from which η_u can be determined with good accuracy. A similar calculation can be carried out to obtain $p_v(v)$, and hence η_v, from the other set of data.

3.3 Data-reduction using Fourier transforms

Clearly the most satisfactory experimental technique is to arrange where-ever possible that there are very many fringes contained within the beam diameter, since analysis can then proceed with the minimum of *a priori* assumptions about the flow. In addition, we have seen in section 3.2.2(a) that the problem of data-reduction collapses to that of carrying out a Fourier transform on the experimental data, which is a simple and rapid computational procedure. We remark here that the fast Fourier transform (FFT) is not usually the appropriate technique, since it lacks flexibility and, at low turbulence levels, the necessary resolving power. However, it has been successfully used in difficult engineering applications[9], and if sufficient signal is available from each scatterer, so that the correlator can be operated in a burst mode, the FFT is the obligatory method for processing each autocorrelation function.

We have also seen that the Fourier transform method can be adapted to cover experiments on high-speed flows where the turbulence levels are fairly low, which includes many of the types of flow of particular interest at the Royal Aircraft Establishment. For these reasons, our efforts in the field of data-reduction have been concentrated mainly on the development of Fortran programmes based on this technique. A detailed description of the structure of such a programme, and of the hardware used for on-line applications, is given in another paper presented at this conference[10]. We now discuss some important characteristics of the Fourier transform in laminar and low-turbulence flows.

3.3.1 Laminar flows

The limited number of channels available in a clipping correlator significantly restricts the number of cycles available in the record obtained from a laminar flow. The rectangular data-window produces an effect in the transform plane which is exactly equivalent to instrumental broadening in classical spectroscopy. The delta-function which would ideally be present is replaced by a function having a sin x/x behaviour, although still centred at the same point on the u-axis. Thus an accurate estimation of the velocity can be made simply by finding the position of the peak of the Fourier transform. (We ignore the extremely small perturbation which will arise from the associated sin x/x function centred at the symmetric point on the negative u-axis.) We shall now show that the sin x/x structure can also be used to decide whether, within the limits of experimental accuracy, the flow is laminar.

We assume that, if present, the beam profile term (the exponential factor) has been removed in the manner described in section 3.2.2(b) together with any residual constant term. Hence we have an equation of the form

$$G(\tau) = c \int_0^\infty p_u(u) \cos \frac{2\pi u\tau}{s}\, du \tag{17}$$

where c is some constant. The inverse transform is, by the Wiener-Khinchine theorem,

$$p_u(u) = \frac{2}{\pi c} \int_0^\infty G(\tau) \cos \frac{2\pi u\tau}{s}\, d\tau \quad . \tag{18}$$

Now for laminar flow at velocity u_0 ,

$$G(\tau) = \cos \frac{2\pi u_0 \tau}{s} \quad .$$

The data record terminates at some value τ_m , say, of τ , so that the computed function will be, writing ω and ω_0 for $2\pi u/s$ and $2\pi u_0/s$ respectively.

$$p_u(u) = \frac{2}{\pi c} \int_0^{\tau_m} \cos \omega_0\tau \cos \omega\tau\, d\tau \tag{19}$$

or

$$p_u(u) = \frac{1}{\pi c}\left[\frac{\sin\{(\omega - \omega_0)\tau_m\}}{\omega - \omega_0} + \frac{\sin\{(\omega + \omega_0)\tau_m\}}{\omega + \omega_0}\right] \quad .$$

At $u = u_0$, the ratio of the first term to the second is at least $2\omega_0\tau_m$. But the time per cycle is $2\pi/\omega_0$, so that the number of cycles contained in the record, say q , is $\omega_0\tau_m/2\pi$ and the ratio is at least $4\pi q$. A typical experimental value for q , using a 48-channel correlator, would be about 8, so that the first term would be more than 100 times the maximum value of the second at resonance. Hence around $u = u_0$ we take as a good approximation

$$p_u(u) = \frac{\tau_m}{2\pi^2 c} \, \frac{\sin\left\{2\pi q\left(\frac{u}{u_0} - 1\right)\right\}}{q\left(\frac{u}{u_0} - 1\right)} \quad ,$$

in terms of q .

The first zeros on either side of the peak at $u = u_0$ occur when

$$2\pi q\left(\frac{u}{u_0} - 1\right) = \pm \pi$$

ie when

$$u = u_0\left(1 \pm \frac{1}{2q}\right) .$$

Suppose we regard the curve between these two points as defining a velocity distribution. Then the 'turbulence intensity' (the ratio of the standard deviation to the mean) associated with such a distribution, say ψ, would be given by the formula

$$\psi = \frac{1}{u_0}\left[\frac{\int_{\alpha_-}^{\alpha_+} (u - u_0)^2 p_u(u)\,du}{\int_{\alpha_-}^{\alpha_+} p_u(u)\,du}\right]^{\frac{1}{2}}$$

where $\alpha_+ = u_0\left(1 + \frac{1}{2q}\right)$, $\alpha_- = u_0\left(1 - \frac{1}{2q}\right)$.

Putting

$$x = 2\pi q\left(\frac{u}{u_0} - 1\right)$$

we find

$$\psi = \frac{1}{q\sqrt{2\pi}}\left(\int_{-\pi}^{\pi} \frac{\sin x}{x}\,dx\right)^{-\frac{1}{2}} . \qquad (20)$$

From tables,

$$\psi \triangleq \frac{1}{4.82q} .$$

Hence ψ will depend purely on the number of cycles in the record. The programme can easily be arranged to determine this quantity as a by-product of other calculations, and it can be compared with the estimate of turbulence intensity generated by the main programme. (This searches outwards from the peak in the Fourier transform plane and also uses as limits for the integration the points at which the distribution first crosses the u-axis .) If the two values agree closely, it can be concluded that the flow is essentially laminar; any turbulence present in the flow would result in an estimate for turbulence intensity greater than ψ .

3.3.2 Flows with low turbulence levels

At sufficiently high turbulence levels, the autocorrelation function will have decayed away effectively to zero by the end of the record; for a 48-channel correlator operating with typical settings on a flow having a fairly symmetric unimodal velocity distribution, the level below which truncation effects begin to appear is about 4 or 5%. Mean velocity estimates are very little affected at such low levels, but the accurate determination of the turbulence intensity begins to pose problems. If a good model of the flow is available for curve-fitting purposes, it should be possible to obtain accurate estimates down to the laminar condition; for example, a Gaussian model would often be acceptable. Methods for extrapolation of the autocorrelation function, using analytic continuation[11] or maximum entropy[12] techniques, have also been proposed. However, they would all require a separate and distinct programme. A procedure has therefore been devised, based on an extension of the technique described in the last section, which uses information already available in the Fourier transform plane.

The basic assumption is that at these low turbulence levels the distributions are effectively Gaussian. For each value of turbulence intensity η_t over the desired region (typically 0 - 5%), an autocorrelation function having a specified length of q cycles can be computed; this represents the data that would be obtained in an ideal experiment on such a flow. Since the autocorrelation data are truncated, the Fourier transform will consist of a broadened central lobe with oscillatory wings. From this central lobe, defined by the points at which the curve first crosses the u-axis on either side of the peak, an apparent turbulence intensity η_a can be calculated. It is found as a matter of experiment that if, for various values of q , the quantity $q\eta_a$ is plotted against $q\eta_t$, the curves virtually coincide, and a single look-up table can therefore be constructed to cover a range of values of the product $q\eta_a$. Estimates of q and η_a are available from the main programme and the refined estimate of turbulence intensity, η_t , can be made part of the output data-set.

3.4 The autocorrelation function for a single transit in the presence of noise

It has been remarked previously that an experimental autocorrelation function will always contain a reasonably constant additive term, usually arising mainly from the stray radiation present in the experimental environment. If it can be assumed that only one scatterer has contributed to the data, interesting information about the circumstances of the experiment can be extracted in the following way.

Equation (4) can be written, ignoring the phase constant $(2\pi/s)y_0$,

$$I(t) = c_1\left[\exp\left\{-\frac{2(u^2+v^2)t^2}{r_0^2}\right\}\right]\left(1 + m\cos\frac{2\pi ut}{s}\right) + c_2 \tag{21}$$

where $c_1 = I_0(1+\rho^2)\exp\left\{-\frac{2}{r_0^2}\left(y_0^2 + z_0^2\right)\right\}$

and c_2 is the intensity of the background illumination, which is assumed to be constant. *Then if the experiment lasts for a time* T, long compared with the beam transit time $r_0/\sqrt{(u^2+v^2)}$, we have

$$G(\tau) = \int_{-T/2}^{T/2} I(t)I(t+\tau)dt$$

$$= c_1^2\frac{\sqrt{\pi}}{2}\frac{r_0}{\sqrt{u^2+v^2}}\left[\exp\left\{-\frac{(u^2+v^2)\tau^2}{r_0^2}\right\}\right]\left(1 + \tfrac{1}{2}m^2\cos\frac{2\pi u\tau}{s}\right) + c_1c_2\sqrt{2\pi}\frac{r_0}{\sqrt{u^2+v^2}}$$

$$\times\left[1 + \tfrac{1}{2}m\exp\left\{-\frac{\pi^2r_0^2u^2}{2s^2(u^2+v^2)}\right\}\right] + c_2^2T .$$

The second exponential term will be quite negligible in any real experiment and we can write

$$G(\tau) = c_1^2\frac{\sqrt{\pi}}{2}\frac{r_0}{\sqrt{u^2+v^2}}$$

$$\times\left[\exp\left\{-\frac{(u^2+v^2)\tau^2}{r_0^2}\right\}\right]\left(1 + \tfrac{1}{2}m^2\cos\frac{2\pi u\tau}{s}\right) + c_1c_2\sqrt{2\pi}\frac{r_0}{\sqrt{u^2+v^2}} + c_2^2T .$$

$$\ldots\ldots \tag{22}$$

(Note that the units of $I(t)$ are energy (pulses) per unit time and of $G(\tau)$, energy squared per unit time.)

Suppose the maximum and minimum channel contents near the origin $(\tau = 0)$ are G_{max} and G_{min} - these correspond to values of the cosine term of approximately +1 and -1 respectively. Then if τ_s is the sample time, *ie* the width of a channel, and if m is taken to be unity,

$$\frac{G_{max} - G_{min}}{\tau_s} \simeq c_1^2 \frac{\sqrt{\pi}}{2} \frac{r_0}{\sqrt{u^2 + v^2}}$$

from which c_1 can be found. Also

$$G(0) = \frac{G_{max}}{\tau_s} = c_1^2 \frac{3\sqrt{\pi}}{4} \frac{r_0}{\sqrt{u^2 + v^2}} + c_1 c_2 \sqrt{2\pi} \frac{r_0}{\sqrt{u^2 + v^2}} + c_2^2 T ,$$

giving c_2.

The total number of pulses received by the correlator is

$$N_{tot} = \int_{-T/2}^{T/2} I(t)dt \approx c_1 \sqrt{\frac{\pi}{2}} \frac{r_0}{\sqrt{u^2 + v^2}} + c_2 T \quad . \tag{23}$$

Hence the number of signal counts is

$$N_s = c_1 \sqrt{\frac{\pi}{2}} \frac{r_0}{\sqrt{u^2 + v^2}}$$

and the number of background counts is

$$N_b = c_2 T \quad .$$

In Ref 10 a correlation function acquired over a period of 25 ms is displayed. For this experiment, the relevant values are found to be, approximately, $G_{max} = 22$, $G_{min} = 11$. The sample time setting τ_s was 0.25 μs, the beam radius about 500 microns and the velocity across the beam about 33 m s^{-1}. We find for this case

$$\begin{aligned} c_1 &= 1.819\ \mu s^{-1} \\ c_2 &= 0.0283\ \mu s^{-1} \\ N_s &= 34 \\ N_b &= 708 \quad . \end{aligned}$$

The peak count rate would have been, putting $t = 0$ and $m = 1$ in (21), 3.67 MHz, while the background count rate was 28.3 kHz. It is interesting to note that a good velocity estimate appears to be obtainable from only 34 signal pulses, in the presence of considerable noise.

4 CONCLUDING REMARKS

It has been demonstrated that the characteristics of the measuring volume make some degree of approximation in the interpretation of the Doppler-difference signal or of its autocorrelation function unavoidable, and that, in the absence of a complete model for the velocity field of the fluid under study, further simplifying assumptions have to be made. The errors which result can be made small, however, by careful design of the experiment. Specifically, the optical geometry should be arranged wherever possible so that the period of a Doppler-difference cycle is very much less than the beam radius transit time; this case involves the smallest number of assumptions about the nature of the flow. In addition, it has been shown that the data-reduction procedure is then of the simplest kind.

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REFERENCES

No.	Author	Title, etc
1	H.Z. Cummins E.R. Pike (eds)	*Photon correlation and lightbeating spectroscopy.* Plenum Press, New York (1974)
2	J.B. Abbiss	Photon correlation velocimetry in aerodynamics. In *Photon correlation spectroscopy and velocimetry*, edited by H.Z. Cummins and E.R. Pike, pp 386-424, Plenum Press, New York (1977)
3	L.J.S. Bradbury	*Proceedings, Second International Conference on Photon Correlation Techniques in Fluid Mechanics*, Stockholm, Sweden, 14-16 June 1978
4	L.F. East	Private communication.
5	A.D. Birch D.R. Brown M.G. Dodson J.R. Thomas	Photon correlation for the determination of mixtures and velocity fields in aerodynamic systems. *First Conference on Photon Correlation Techniques in Fluid Mechanics*, Churchill College, Cambridge, England, 6-7 April 1977
6	J.G. McWhirter E.R. Pike	As Ref 3
7	F. Durst A. Melling J.H. Whitelaw	*Principles and practice of laser-Doppler anemometry*, pp 144-151. Academic Press, London (1976)
8	J.B. Abbiss L.J.S. Bradbury M.P. Wright	Measurements in an axisymmetric jet using a photon correlator. *Symposium on Laser-Doppler Anemometry.* Technical University of Denmark, Copenhagen, August 1975
9	C.J. Moore A.E. Smart	*J. Phys. E (GB)* 9, 977-981 (1976)
10	P.R. Sharpe	As Ref 3
11	B.R. Frieden	*J. Opt. Soc. Amer.* 57, 8, 1013-1019 (1976)
12	P.P. Stone L.J.S. Bradbury	As Ref 3

Fig 1

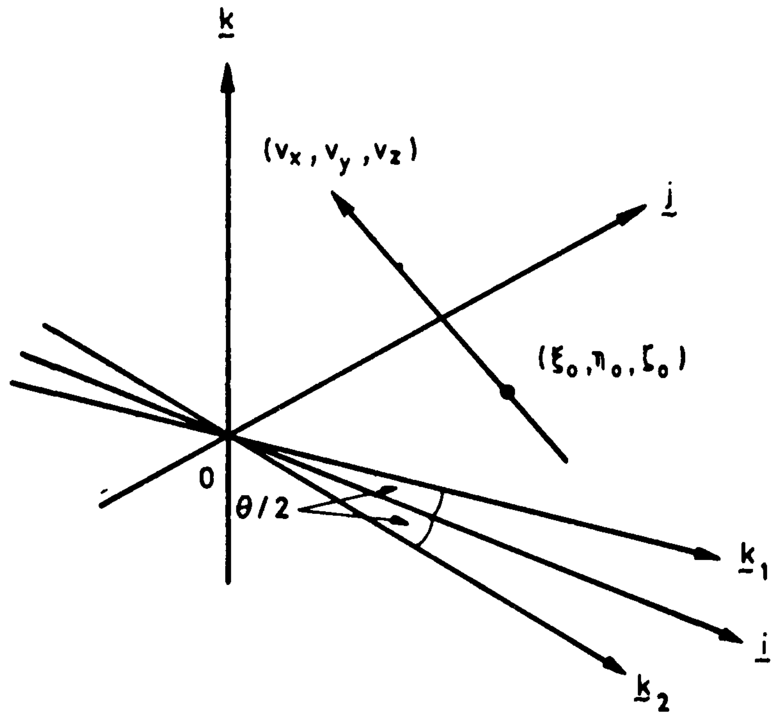

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Fig 1 Scattering geometry

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17. Abstract

The Doppler-difference optical arrangement is nowadays the one most commonly used for fluid flow investigations with a laser anemometer. In this review, the essential characteristics of the Doppler-difference signal and of the associated autocorrelation function are considered for both laminar and turbulent flows. Experimental conditions under which it is known that the autocorrelation function can be analysed to yield reliable estimates of mean velocity and turbulence intensity are specified.

The various data-reduction methods which have been proposed are classified and briefly reviewed. It is shown that unknown flows having relatively high levels of turbulence can only be treated successfully if there are very many fringes within a beam diameter, or if frequency-shifting techniques are used to achieve an equivalent effect. The estimation of turbulence intensity in low-turbulence flows also presents special difficulties and a procedure which uses information already available in the transform plane is described.